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Research Article

ENERGY OF SOME DERIVED GRAPHS

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Abstract

In this article we establish energy of derived graphs, by using adjacency distance matrix of derived graph. This article contains energy of derived graphs of some standard graphs such as; star graph, double star, complete bipartite graph, crown graph, friendship graph, healthy spider graph and cocktail party graph. Also, we determine lower bound and upper bound of energies of derived graphs.

Keywords: Derived graph, Derived Matrix, Derived.

INTRODUCTION

I Gutman was introduced a very useful concept, that is the energy of a graph in the year 1978 [7]. Let G be a simple and connected graph with n vertices and m edges and let $A = (a_{ij})$ be the real symmetric adjacency square matrix of order $n \times n$ of the graph G with real eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ and there sum is equal to zero. The energy of a graph G is given by;

$$\xi(G) = \sum_{i=1}^{n} |\lambda_i|.$$

Theories on the mathematical concepts of graph energy can be seen in [2, 3, 4, 5, 6, 10, 15] and the references cited there in. For various upper and lower bounds for energy of a graph can be found in papers [4, 6] and it was observed that graph energy has chemical applications in the molecular orbital theory of conjugated molecules [8].

Derived Graph

In this paper, we consider simple graphs, which are the graphs without directed, multiple, or weighted edges, and without self-loops. Let G be such a graph and let its vertex set be $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set be $E(G) = \{e_1, e_2, \ldots, e_m\}$. The distance between the vertices v_i and v_j is equal to length of a shortest path between v_i and v_j.

Definition 1.1. Let the derived graph is also a simple graph denoted by G_D, is the graph having the same vertex set as G, in which two vertices are adjacent if and only if their distance in G is 2 [16].

Energy of Derived Graph

Definition 1.2. The adjacency distance matrix of derived graph GD of order $n \times n$ is defined by,

$A(G_D) = (d_{ij})$, where

$$(d_{ij}) = \begin{cases} 1 \text{ if } \mathrm{d(i,j)} = 2, \text{where } \mathrm{d(i,j)} \text{ is distance between i and j} \\ 0 \quad \text{if} \quad \text{Otherewise.} \end{cases}$$

The characteristic polynomial of $A(G_D)$ is defined as; $f_n(G_D, \alpha) = det(A(G_D) - \alpha I)$.

The characteristic equation is; $det(A(G_D) - \alpha I) = 0$.

Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the derived eigenvalues of $A(G_D)$. The energy of derived graph G_D is defined as;

$$\xi(G_D) = \sum_{i=1}^{n} |\alpha_i|.$$

RESULTS

Definition 2.1. A graph G with $\{v_1, v_2, \ldots, v_n\}$ as a set of vertices in which $v_2, v_3, \ldots, v_n$ are connected to a common vertex v_1 which has a degree $n - 1$, denoted by $K_{1,n}$[9].

Theorem 2.2. The energy of derived star graph, $K_{D(1,n)}$ is $2(n - 1)$.

Proof. The adjacency matrix of $K_{D(1,n)}$ is given by;

$$\mathrm{A_D}[K_{D(1,n)}] = \begin{bmatrix} 0\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 0\,0\,1 \ldots\ldots\ldots 1\,1\,1 \\ 0\,1\,0 \ldots\ldots\ldots 1\,1\,1 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0\,1\,1 \ldots\ldots\ldots 0\,1\,1 \\ 0\,1\,1 \ldots\ldots\ldots 1\,0\,1 \\ 0\,1\,1 \ldots\ldots\ldots 1\,1\,0 \end{bmatrix}_{(n+1)\times(n+1)}$$

The Characteristic polynomial is:$\lambda[\lambda - (n - 1][\lambda + 1]^{n-1}$.

The Characteristic equation is: $\lambda[\lambda - (n - 1)][\lambda + 1]^{n-1} = 0$.

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Then,

$$\mathrm{Spec_D}\left[K_{D(1,n)}\right] = \begin{pmatrix} 0 & n-1 & -1 \\ 1 & 1 & n-1 \end{pmatrix}.$$

Hence the energy of $K_{D(1,n)}$ is;

$$\xi_D\left(K_{D(1,n)}\right) = |0|.1 + |n-1|.1 + |-1|.(n-1) = 2(n-1).$$

Definition 2.3. A tree containing exactly two non-pendent vertices is called double star, denoted by $S_{(m,n)}$ [14].

Theorem 2.4. The energy of derived double star graph, $S_{D(m,n)}$ is $2(m+n)$.

Proof. The adjacency matrix of $S_{D(m,n)}$ is given by;

$$\mathrm{A_D}[S_{D(m,n)}] = \begin{bmatrix} 0\,0\,0 \ldots\ldots\ldots 1\,1\,1 \\ 0\,0\,1 \ldots\ldots\ldots 0\,0\,0 \\ 0\,1\,0 \ldots\ldots\ldots 0\,0\,0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 1\,0\,0 \ldots\ldots\ldots 0\,1\,1 \\ 1\,0\,0 \ldots\ldots\ldots 1\,0\,1 \\ 1\,0\,0 \ldots\ldots\ldots 1\,1\,0 \end{bmatrix}_{(m+n+2)\times(m+n+2)}$$

The Characteristic polynomial is: $[\lambda - m][\lambda - n][\lambda + 1]^{m+n}$.

The Characteristic equation is: $[\lambda - m][\lambda - n][\lambda + 1]^{m+n} = 0$.

Then,

$$\mathrm{Spec_D}\left[S_{D(m,n)}\right] = \begin{pmatrix} m & n & -1 \\ 1 & 1 & m+n \end{pmatrix}.$$

Hence the energy of $S_{D(m,n)}$ is;

$$\xi_{MD}\left(S_{D(m,n)}\right) = |m|.1 + |n|.1 + |-1|.m+n = 2(m+n).$$

Definition 2.5. A complete bipartite graph $K_{m,n}$ is a graph in which each vertex in set A is adjacent to each vertex in set B, with $|A| = m$ and $|B| = n$ [9].

Theorem 2.6. The energy of derived complete bipartite graph, $K_{D(m,n)}$ is $2(m+n-2)$.

Proof. The adjacency matrix of $K_{D(m,n)}$ is given by;

$$\mathrm{A_{MD}}[K_{D(m,n)}] = \begin{bmatrix} 0\,1\,1 \ldots\ldots\ldots 0\,0\,0 \\ 1\,0\,1 \ldots\ldots\ldots 0\,0\,0 \\ 1\,1\,0 \ldots\ldots\ldots 0\,0\,0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 0\,0\,0 \ldots\ldots\ldots 0\,1\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,0\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,1\,0 \end{bmatrix}_{(m+n)\times(m+n)}$$

The Characteristic polynomial is: $[\lambda - (m-1)][\lambda - (n-1)][\lambda + 1]^{m+n-2}$.

The Characteristic equation is: $[\lambda - (m-1)][\lambda - (n-1)][\lambda + 1]^{m+n-2} = 0$.

Then,

$$\mathrm{Spec_D}\left[K_{D(m,n)}\right] = \begin{pmatrix} m-1 & n-1 & -1 \\ 1 & 1 & m+n-2 \end{pmatrix}.$$

Hence the energy of $K_{D(m,n)}$ is;

$$\xi_D\left(K_{D(m,n)}\right) = |m-1|.1 + |n-1|.1 + |-1|.m+n-2 = 2(m+n-2).$$

Definition 2.7. A graph in which any two distinct vertices are adjacent is called complete graph, K_n [9].

Theorem 2.8. Let K_{Dn} be the derived graph of complete graph. Then $\xi_D(K_{Dn}) = 1$.

Definition 2.9. A Crown graph Cr_n is a graph obtained by complete bipartite graph $K_{n,n}$ by removing a perfect matching [13].

Theorem 2.10. Let Cr_{Dn} be the derived graph of crown graph. Then the energy of Cr_{Dn} is $4(n-1)$.

Proof. The adjacency matrix of Cr_{Dn} is given by;

$$\mathrm{A_D}[Cr_{Dn}] = \begin{bmatrix} 0\,1\,1 \ldots\ldots\ldots 0\,0\,0 \\ 1\,0\,1 \ldots\ldots\ldots 0\,0\,0 \\ 1\,1\,0 \ldots\ldots\ldots 0\,0\,0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 0\,0\,0 \ldots\ldots\ldots 0\,1\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,0\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,1\,0 \end{bmatrix}_{(2n)\times(2n)}$$

The Characteristic polynomial is: $[\lambda - (n-1)]^2[\lambda + 1]^{2(n-2)}$.

The Characteristic equation is: $[\lambda - (n-1)]^2[\lambda + 1]^{2(n-2)} = 0$.

Then,

$$\mathrm{Spec_D}[Cr_{Dn}] = \begin{pmatrix} n-1 & -1 \\ 2 & 2(n-1) \end{pmatrix}.$$

Hence the energy of Cr_{Dn} is;

$$\xi_D(Cr_{Dn}) = |n-1|.2 + |-1|.2\,(n-1) = 4(n-1)$$

Definition 2.11. A friendship graph F_n is a graph in which n number of triangles are connected in common central vertex [12].

Theorem 2.12. The energy derived friendship graph F_{Dn} is $4(n-1)$.

Proof. The adjacency matrix of F_{Dn} is given by;

$$\mathrm{A_D}[F_{Dn}] = \begin{bmatrix} 0\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 0\,0\,0 \ldots\ldots\ldots 1\,1\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,1\,1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 0\,0\,0 \ldots\ldots\ldots 0\,1\,1 \\ 0\,0\,0 \ldots\ldots\ldots 1\,0\,0 \\ 0\,0\,0 \ldots\ldots\ldots 1\,0\,0 \end{bmatrix}_{(2n+1)\times(2n+1)}$$

The Characteristic polynomial is: $\lambda^{n+1}[\lambda - 2(n-1)][\lambda + 2]^{n-1}$.

The Characteristic equation is: $\lambda^{n+1}[\lambda - 2(n-1)][\lambda+2]^{n-1} = 0$.

Then,

$$\mathrm{Spec_D}[F_{Dn}] = \begin{pmatrix} 0 & 2(n-1) & -2 \\ n+1 & 1 & n-1 \end{pmatrix}.$$

Hence the energy of F_{Dn} is;

$$\xi_D(F_{Dn}) = |0|.(n+1) + |2(n-1)|.1 + |-2|.(n-1) = 4(n-1).$$

Definition 2.13. Let G be a Healthy spider graph with $2n+1$ vertices and $2n$ edges, denoted by Hs_n[1].

Theorem 2.14. Let Hs_{Dn} be the derived graph of healthy spider graph. Then the energy ofHs_{Dn} is $2(n-1) + \sqrt{n}$.

Proof. The adjacency matrix of Hs_{Dn} is given by;

$$\mathrm{A_D}[Hs_{Dn}] = \begin{bmatrix} 0\,0\,0 \ldots\ldots\ldots 1\,1\,1 \\ 0\,0\,1 \ldots\ldots\ldots 0\,0\,0 \\ 0\,1\,0 \ldots\ldots\ldots 0\,0\,0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 1\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 1\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 1\,0\,0 \ldots\ldots\ldots 0\,0\,0 \end{bmatrix}_{(2n+1)\times(2n+1)}$$

The Characteristic polynomial is: $\lambda^{(n-1)}[\lambda-(n-1)][\lambda+1]^{n-1}[\lambda^2-n]$.

The Characteristic equation is: $\lambda^{(n-1)}[\lambda-(n-1)][\lambda+1]^{n-1}[\lambda^2-n] = 0$.

Then,

$$\mathrm{Spec_D}[Hs_{Dn}] = \begin{pmatrix} 0 & n-1 & -1 & \sqrt{n}. \\ n-1 & 1 & n-1 & 1 \end{pmatrix}.$$

Hence the of Hs_{Dn} is;

$$\xi_D(Hs_{Dn}) = |0|.(n-1) + |n-1|.1 + |-1|(n-1 + |\sqrt{n}|.1 = 2(n-1) + \sqrt{n}.$$

Definition 2.15. The cocktail party graph is denoted by Cp_n, is a graph having the vertex set $V = S\{u_i, v_i\}$ and the edge set $E = \{u_iu_j,\ v_iv_j : i \neq j\} \cup \{u_iu_j,\ v_iv_j : 1 \leq i < j < n\}$ [13].

Theorem 2.16. The energy derived cocktail party graph Cp_{Dn} is $2n$.

Proof. The adjacency matrix of Cp_{Dn} is given by;

$$\mathrm{A_D}[Cp_{Dn}] = \begin{bmatrix} 0\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 0\,0\,0 \ldots\ldots\ldots 000 \\ 0\,0\,0 \ldots\ldots\ldots 1\,00 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0\,0\,1 \ldots\ldots\ldots 0\,0\,0 \\ 0\,0\,0 \ldots\ldots\ldots 0\,0\,0 \\ 0\,0\,0 \ldots\ldots\ldots 0\,0\,0 \end{bmatrix}_{(2n)\times(2n)}$$

The Characteristic polynomial is: $[\lambda-1]^n[\lambda+1]^n$.

The Characteristic equation is: $[\lambda-1]^n[\lambda+1]^n = 0$.

Then,

$$\mathrm{Spec_D}[Cp_{Dn}] = \begin{pmatrix} 1 & -1 \\ n & n \end{pmatrix}.$$

Hence the energy of Cp_{Dn} is;

$$\xi_D(Cp_{Dn}) = |1|.n + |-1|.n = 2n.$$

Properties of Derived Graphs Energy

Theorem 3.1. Let G_D be a derived graph with vertex set, $V(G_D) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G_D) = \{e_1, e_2, \ldots, e_m\}$. If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of derived graph adjacency matrix $A_{MD}(G_D)$, then

1. $\sum_{i=1}^{n} \lambda_i = 0$
2. $\sum_{i=1}^{n} \lambda^2{}_i = 2M$

Proof. 1. We know that, the sum of spectra of$A_D(G_D)$ is zero.

$\therefore$

$\sum_{i=1}^{n} \lambda_i = 0$.

2. We have,

$$\sum_{i=1}^{n} \lambda^2{}_i = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}d_{ij} = \sum_{i=1}^{n} d_{ii}{}^2 + \sum_{i\neq j}^{n} d_{ij}{}^2 = 0 + 2\sum_{i\leq j}^{n} d_{ij}{}^2$$

$= 2M$.

Where $\sum_{i\leq j}^{n} d_{ij}{}^2 = 2M$.

Bounds for Derived Graphs Energy

Similar to McClellands [11] bounds for energy of a graph, bounds for $\xi(G_D)$ are given in the following theorem;

Theorem 4.1. Let G_D be a derived graph of order n and size m. If $P = \prod_{i=1}^{n} |\lambda|^{2n}$, then $\sqrt{(2M + n(n-1)P)} \leq \xi_D(G_D) \leq \sqrt{(2Mn)}$.

Proof. We have Cauchy Schwarz inequality, which is;

$$(\sum_{i=1}^{n} a_ib_i)^2 \leq \sum_{i=1}^{n}(a_i)^2 \sum_{i=1}^{n}(b_i)^2$$

Let $a_i = 1$ and $b_i = |\lambda_i|$

Hence,

$$(\sum_{i=1}^{n} 1.|\lambda_i|)^2 \leq \sum_{i=1}^{n}(1)^2.\sum_{i=1}^{n}(|\lambda_i|)^2$$

$$(\sum_{i=1}^{n} |\lambda_i|)^2 \leq n\sum_{i=1}^{n}(|\lambda_i|)^2$$

$$\xi_D(G_D) \leq n(2M). \qquad (1)$$

Next, we know that

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq \prod_{i\neq j}(|\lambda_i||\lambda_j|)^{\frac{1}{n(n-1)}}$$

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq \prod_{i\neq j}(|\lambda_i|)^{2(n-1)\frac{1}{n(n-1)}}$$

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq \prod_{i\neq j}(|\lambda_i|)^{\frac{2}{n}} \qquad (2)$$

Put $\prod_{i\neq j}(|\lambda_i|)^{\frac{2}{n}} = P$

∴ Equation 2 becomes;

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq P$$

$$\sum_{i\neq j}|\lambda_i||\lambda_j| \geq n(n-1)\,P$$

Now,

$$\xi_D(G_D)^2 = \sum_{i=1}^{n}(|\lambda_i|)^2$$

$$= \sum_{i=1}^{n}(|\lambda_i|)^2 + \sum_{i\neq j}|\lambda_i||\lambda_j|$$

$\xi_D(G_D)^2 \geq 2M + n(n-1)P (4.3)$

Hence from 4.1 and 4.3 we have,

$\sqrt{(2M + n(n-1)P)} \leq \xi_D(G_D) \leq \sqrt{(2Mn)}$.

Theorem 4.2. If λ_1 is the largest eigenvalue of $A_D(G_D)$, then $\lambda_1 \geq \frac{2M}{n}$.

Proof. Let X be any non-zero vector. Then by [15], We have $\lambda_1(A) = \max_{X\neq 0}\left\{\frac{X'AX}{X'X}\right\}$.

$\therefore \lambda_1 \geq \frac{J'AJ}{J'J} = \frac{2M}{n}$, where J is a unit matrix $[1, 1, 1, \ldots, 1]$.

Similar to Koolen and Moultons [10] upper bound for energy of a graph, upper bound for ξ_{MD} is given in the following theorem.

Theorem 4.3. If G_D is a derived graph with n vertices and m edges and $K + 2M \geq n$, then $\xi_D(G_D) \leq \frac{2M}{n} + \sqrt{((n-1)[2M - \left(\frac{2M}{n}\right)^2]}$.

Proof. We have Cauchy Schwarz inequality,

$$(\sum_{i=1}^{n} a_i b_i)^2 \leq \sum_{i=1}^{n}(a_i)^2 \sum_{i=1}^{n}(b_i)^2$$

Let $a_i = 1$ and $b_i = |\lambda_i|$

Hence,

$$(\sum_{i=1}^{n} 1.|\lambda_i|)^2 \leq \sum_{i=1}^{n}(1)^2.\sum_{i=1}^{n}(|\lambda_i|)^2$$

$$[\xi_D(G_D) - \lambda_1]^2 \leq (n-1)[2M - {\lambda_1}^2]$$

$\xi_D(G_D) \leq \lambda_1 + \sqrt{[(n-1)[2M - {\lambda_1}^2]]}$.

Let $f(x) = x + \sqrt{[(n-1)[2M - x^2]]}$

For decreasing function $f'(x) \leq 0$

$\Rightarrow 1 - \frac{(n-1)x}{\sqrt{[(n-1)[2M - x^2]]}} \leq 0$

$\Rightarrow x \geq \sqrt{[\frac{2M}{n}]}$

Since $2M \geq n$, we have $\sqrt{[\frac{2M}{n}]} \leq \frac{2M}{n} \leq \lambda_1$

$\therefore f(x) \leq f(\frac{2M}{n})$

That is, $\xi_D(G_D) \leq f(\lambda_1) \leq f(\frac{2M}{n})$

$\Rightarrow \xi_D(G_D) \leq f(\frac{2M}{n})$

$\Rightarrow \xi_D(G_D) \leq \frac{2M}{n} + \sqrt{((n-1)[2M - \left(\frac{2M}{n}\right)^2]}$.

Conclusion

In this article we established energy of derived graphs of some standard graphs like star graph, double star, complete bipartite graph, crown graph, friendship graph, healthy spider graph and cocktail party graph. Also, we determined upper bound and lower bound of energy of derived graphs.

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