



Research Article

A MISCELLANEOUS CONCLUSION & SOME LIMITATIONS FOR THE HKLAM STATISTICAL MODEL THEORY

*Kai Shun, Lam

Department of Mathematics, Science Faculty, University of Hong Kong, Hong Kong

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Abstract

There may be numerous discussions about my HK-Lam Statistical Model theory. In practice, the theory is about turning any giving random matrix into a linear combination through some suitable linear transformation. Through the linear combination, we may then convert it into the linear regression (without reality engineering error term). In order to fill the gap between pure theory and the apply engineering, one may then use the machine learning (e.g. gradient descent) method to find out the expected error term. At the same time, for the inverse of the HKLam Statistical Model Theory, there may be a $1/x \rightarrow \text{infinity}$ situation, this author thus proposes a primal duality gap ($\delta > 0, = 0, < 0$) cases to solve such inverse problem (or alternatively, in the mathematical geometric sense, by definition, the inverse of the linear transformation is just a constant k multiplied with a normal unit vector i.e. $\underline{x}^{-1} = (\underline{x} / \|\underline{x}\|)^2 = (1/\|\underline{x}\|) * (\underline{x}/\|\underline{x}\|) = k*\underline{u}$ and $k = (1/\|\underline{x}\|)$ and $\|\underline{x}\|$ NOT equal to zero. Otherwise, we have the fact that $A\underline{0} = \underline{0}$ is the only trivial solution and thus the unit normal unit vector will definitely NOT equal or even exclude the case of a zero vector. In fact, $k = \|A(\underline{x}_1, \underline{x}_2, \dots, \underline{x}_n)^{-1}(b_1, b_2, \dots, b_n)^{-1}\|$. Hence, from the inverse HK-Lam theory, we may find some more hidden social discussing issues or implications from the final Nash Equilibrium matrix and thus we may further make those essential and necessary improvements, policy and contributions to our society.

Keywords: HKLam, Engineering.

INTRODUCTION

It is no doubt that there may be lots of discussion in concerning the social credit (information) system (SCIS). People are usually worrying about the monitoring related actions that may associated with the SCIS especially the present non-democratic Eastern countries like Vietnam, China and North Korea etc. Indeed, the credit system is now heatly tested especially in China. Certainly, there are some people who welcomes much for the system as they may do their voluntee works during their leisure time and as a reward for a case in obtaining a discount to transportation. However, for those who are just reflecting the blackside of the society, they may suffer very much as they may even need to walk to their working place when the reward marks fall into some negative values. In the Western countries, most of the concerns about the elementray SCIS are in the issue of privacy as the system was usually tested in the commercial fields like in the marketing of products etc. Actually, when people are using their credit cards for buying goods, they are spending their money, at the same time, they may getting some reward points for the exchange of airplanes miles. This act may be the elementary idea of the SCIS but certainly, people can buy the airplane ticket directly by their own cash. In the school, the grade scale mapping system as stated in [] may be another informal type of SCIS.

A Brief Review of the HKLam Statistical Model Theory

Indeed, what is a linear transformation? According to [], geometrically consider the case for a 3-dimensional space, one may take any three basis vectors (like the

$\underline{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\underline{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\underline{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ around the origin on the x-axis,

y-axis, and z-axis that can span the whole 3-dimensional vector space. Indeed, these three basis vectors may then be extented by some multiples, addition, rotation, mirroring and movement etc to form another linear combined vector.

Hence, we may define (in general) geometrically, the above vector-space spanning actions for these basis vectors as the linear transformation (where the output is just the linear combination of these basis vectors and thus is the difference from any arbitrary transformation []) in the following ways:

1. $T(\underline{x} + \underline{y}) = T(\underline{x}) + T(\underline{y})$ and
2. $T(\alpha \underline{x}) = \alpha T(\underline{x})$.

Or a matrix transformation (stretch and flip / linear combination of the matrix basis vectors) as below:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = x_1 \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + x_3 \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

But the vector $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ may already have its linear regression model or its own structural equation,

$$\text{i.e. } \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \beta_0 + \beta_1 x'_{i1} + \beta_2 x'_{i2} + \dots + \varepsilon_i = \mathbf{x}'_i \beta + \varepsilon_i$$

for $i = 1, 2, \dots, n$

$$\text{Or } x_1 \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + x_3 \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} = \beta_0 + \beta_1 x'_{i1} + \beta_2 x'_{i2} + \dots + \varepsilon_i = \mathbf{x}'_i \beta + \varepsilon_i$$

Or for any random square matrix with a suitable transformation, we may always express its matrix

*Corresponding Author: Kai Shun, Lam,

Department of Mathematics, Science Faculty, University of Hong Kong, Hong Kong

transformation vector as a corresponding linear regression model or a structural equation model just like below:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1 \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + x_2 \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + x_3 \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$= \beta_0 + \beta_1 x'_{i1} + \beta_2 x'_{i2} + \dots + \varepsilon_i = x'_i{}^T \beta + \varepsilon_i$$

This author named the above relationship as the HKLam statistical model theory.

As a remark, the mirror image inverse of the above theory is also true as one may find back the random matrix from the corresponding linear regression model equation or the structural model equation by a reversing processing. Theoretically, the HKLam statistical model theory may be stated as:

“If-part”:

For any given squared (or the rectangular) matrix, there must be a suitable matrix transformation such that the resulted matrix transformation vector can be expressed as the linear regression statistical model or a structural equation model;

Mirror Imaged “Only-if part” []:

For any linear regression statistical model or a structural equation model, one may apply its mirror image reverse process in the above “if-part” to find out one of the corresponding & feasible random square (or rectangular) matrix.

$$\text{i.e. } \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}^T \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{-1} \right] = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}^T \frac{1}{|x|}$$

$$\text{where } \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{-1} \right]^T$$

$$\text{and } x'_1 = \frac{x_1}{x_1^2 + x_2^2 + x_3^2}, x'_2 = \frac{x_2}{x_1^2 + x_2^2 + x_3^2}, x'_3 = \frac{x_3}{x_1^2 + x_2^2 + x_3^2}$$

$$\text{Thus } \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}^T \left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^{-1} \right] = \begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = x'_1 y_1 + x'_2 y_2 + x'_3 y_3$$

But $y_1 = y_1 \varepsilon_1$, $y_2 = y_2 \varepsilon_2$ and $y_3 = y_3 \varepsilon_3$ or at least one of the feasible “restored” square matrix, say,

$$\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \text{ or any of the squared (or}$$

rectangular) random matrix (depends on the selection of a suitable set of basis vectors and hence the corresponding linear combination coefficients that may be subjected to the selected basis for the vector $\underline{y}$) where ε_1 , ε_2 , ε_3 are defined as the obvious normal based vectors which has been mentioned in the beginning of the present review section. To be precise,

$\underline{y} = y_1' \underline{v}_1 + y_2' \underline{v}_2 + y_3' \underline{v}_3$ where y_i' is the adjusted linear combination coefficients w.r.t. the selected basis vectors $\underline{v}_i$ and

$$y_i' \underline{v}_i = \begin{pmatrix} a_{1i} \\ a_{2i} \\ a_{3i} \end{pmatrix} \text{ for } i = 1, 2, 3 \text{ etc. Indeed, for the square matrix,}$$

we may also compute the corresponding eigen-values together with the eigen-vectors, find the characteristics polynomial equation as well as the wanted spectral matrix decomposition so that we may solve the system of linear equation in an easier way. By the way, for the rectangular matrix, we may solve the respective system of linear equation by the singular value decomposition. Lastly, the “If-part” of the HKLam statistical model may be reduced into a similar eigen-valued problem such as:

$$(A \underline{x} - \underline{x}^T \beta) = \underline{\varepsilon}$$

i.e. $(A - \beta I) \underline{x} = \underline{0}$ as the starting trivial solution

We may start to search the solution for the above system of linear equation by equating the right-hand side to zero as the trivial solution [] and come across the those vertex points in a (say) 3-dimensional plane by the linear programming method []. Or we may just apply elementary row operation to simplify the linear system []. Then one may obtain the final optimal answer just like the way using in the business operational research method [].

In practice, the HKLam statistical model theory may have its extension for the (Bayesian) Structural Time Series Model [] (which has another significant application in the brain-machine communication interference science and engineering [] & []) together with the rectangular matrices. In reality, we may employ the wireless Wi-Fi charging technique for the implanted integrated circuit during the night sleeping period. Then, the I.C. implanted patient may use the human I.C. - computer interference in the day time for the controlling movement of the mouse symbol over the laptop computer screen such as typing or any other related actions etc. However, the main focus of the present paper is NOT in the real life practice or the engineering way of my papers in [] and []. This author would like to leave the rest of the proofs or the related issues to those interested parties for a further investigation, research and development etc. In brief, most of the details of the HKLam statistical model theory, its proof (if and only if parts) and the description have been written in my previous paper []. This paper only presents the theory’s simplified idea and review as a final summary.

An Elementary Quantitative Verification for the HKLam Statistical Model Theory

To quantitatively verify the truthness of the HKLam statistical model theory, one may refer to my previous paper concerning the novel model of the Riemann Zeta Function’s Root equation. That say, we may first construct a polynomial with the first 10 known values [] of the Riemann Zeta non-trivial roots:

No. of non-trivial Zeta Zeros	Well-known values of the non-trivial Zeta Zeros	No. of non-trivial Zeta Zeros	Well-known values of the non-trivial Zeta Zeros
1	14.1347	6	37.5862
2	21.0220	7	40.9187
3	25.0109	8	43.3271
4	30.4249	9	48.0052
5	32.9351	10	49.7738

Or we may formulate the corresponding interpolation polynomial by the Maple Soft:

$$|(0.0004319 * x^9 - 0.02216 * x^8 + 0.4869 * x^7 - 5.986 * x^6 + 45.1 * x^5 - 214.5 * x^4 + 638.4 * x^3 - 1134 * x^2 + 1080 * x - 395.2)|$$

Next, we may approximate one of the Riemann Zeta Function's Root Model Equation $\frac{\cot x}{\ln(x)}$ by

$$\left(\left(\frac{(-x^2 + 3)}{(3 * x)} \frac{(x + 1)}{(2 * (x - 1))} \right) \right)$$

When we subtract the two equations with each other with the function fsolve in the Maple Soft, then we may get:

The radius of convergence (value) is -0.001257674845 or $|0.001257674845|$ which is greatly smaller than one and is thus convergent. Hence, we may approximate the first 10 non-trivial zeta zeros interpolation polynomial by the model equation or strictly speaking $-\frac{\cot x}{\ln(x)}$ []. In other words, we may conclude (in an analogically way) that both of the controlled teeth model group and investigated teeth model group research once conducted by Dr. Walter Lam & Prof Richard from 2021 to 2023 in the University of Hong Kong, Faculty of Dentistry should be in general be the same. Hence, the causality and my self-developed HKLam statistical model theory have been proved quantitatively or through the Riemann Zeta non-trivial zeros' data. (N.B. This author at that period was once employed as a volunteer – Honorary Research Associate).

In brief, this author have a confidence to prove in a both qualitative (by a pure mathematics) and quantitative (by the first 10 Riemann non-trivial zeta zeros' numerical values) ways that his self-developed and proposed HKLam statistical model theory should be elementary said to be true or correct.

Advantages & Disadvantages of the SCIS for the Reward(s) or Punishment(s)

1. The Advantages of SCIS

To start, people may need a social credit information system because it is easy to create a predictable, fair and transparent business climate. In fact, the benefits of the such system are like:

- Reduce crime rate improve security as the scholar article [] shows that for those cities with high violent crimes, the corresponding credit rating are relatively low. Actually, there may be a relationship between the public management and the credit rating. As the management may reflect how well the willingness of the local government try to pay fully and on time for their debt []. Indeed, the poorer the management of the city local government, the worst of the police security system. This may imply the higher the crime rate. In practice, better local city government management or the social credit as either the rewards or punishments may give raise to a better population affairs control. At the same time, this fact also acts as a good police policy and hence reduce crime rate or a good security.
- Economic development & financial stability as the social credit system may lead to either a reward or punishment, most of the economic development or even the man-power matters can be planned and predicted. This fact may create a further financial stability.
- Consumer Protection as persuading and buying goods may lead to a credits with records and comments. Other buyers may thus have reference from other previous users.

- Civic behaviours and citizen responsibility as those rewards can encourage population to do those positive things while the punishments may act in the mirror image reverse way;

2. The Disadvantages of SCIS

- Less freedom of expression as everywhere is full of control and monitor with doing right for a reward and wrong with a punishment, people are always afraid the loss of freedom and become less aggressive;
- Invasion of Privacy and collection of sensitive information as everything is under the control of the computer information system, people may become more strict and gradually those wrong thing that doing in the private way will become mostly under the sun light. This means there may be less room for doing private things;
- Injustice and discrimination as those punishment may turn into a label of doing things wrongly.

3. Western Business SCIS Vs China SCIS

The origin of the Western social credit system may resort to the last century's 90s when the commercial credit system like the FICO, Equifax and TransUnion tried to facilitate credit assessment in the rural areas together with the small enterprise or business that usually lacks of financial documents. To cite a case, FICO score is indeed a measure of credit risk and has become a fixture of consumer lending in the United States since its beginning as early as mid-1950s. In practice, the main focus of these Western Credit score system company is to provide credit reports and some other fraud-protection products directly the customers. The three major defects of the Western Credit system are:

- Disparate Impacts; B. Arbitrary; C. Opacity

On the other hand, the China social credit system is actually an extension of the existing legal and financial credit rating system in China. Its origin can be traced back to 1980s with the development of a personal banking and financial credit rating system. In reality, the Chinese social credit system is a set of policies and systems which may impact business more than personal such as financial credit reporting and blacklist for for judgement based on specific court orders etc.

4. Ethical considerations for the China SCIS

The China SCIS has been criticized for its control over the personal privacy in every facet of our human life or even the total control over the Chinese population. Indeed, laws may have the possibility to be misused by the government authority for the political purposes or even for an arbitrary abuses.

5. The Best Optimized Improvements Recommended for the SCIS

The major comment of the China type Social Credit Information System is that the everyday normal life will be affected if one's credit becomes negative or attains a minimum. That says, they cannot exchange the necessary and essential daily services through the sufficient amount of rewards. Then they may need to get the service by himself or herself privately but NOT through the public owned services. One may seen such kind of lacking public services as a form of punishment. But indeed, these punishments cannot really

change these group of rebel population. Or such kind of punishments cannot turn the black-sides of the society into a better world. On the contrary, the punishment scheme that associated with the SCIS may in fact increase the social dispute in the society. Finally, the SCIS society may finally approach to the story as depicted in the book named "1984". In order to improve the implementation of the Social Credit Information System, this author suggests that one should NOT ban the right of those rebel group to enjoy the public services, but the government should also employ the commercial buying/selling way of these public services such as the Western credit card reward system. One may buy the wanted services through both of the cash or the reward points from previous spending. Thus, there will NOT be any increase in the disputes between these rebel groups and the government. They will NOT feel to be labelled as the society's outsiders. In practice, the main focus of handling those black-sided issues of the society is the governing authority's willing to listen and looking for any feasible improvements in their policy instead of implementing a blindly punishment system or a Social Credit Information System to punish those who are willing to reflect the blackside of the society.

An (Optional) Large Scale HK-Lam Justification that applied in the SCIS System

In my opinion, there should be an (optional) Large Scale HK-Lam Justification for the SCIS system. But as the SCIS is so controversial and is a new thing, this author suggests that we may need a large-scale test for such kind of the system with the first acknowledged to those volunteered participants. In fact, one may pay much referenced attention to those similar measurements that have been implemented in the commercial business field such as banking marketing credits for those consumers. With the comparison between the ordinary (large scale) case and the commercial banking marketing case, maybe we can get some interested results for the modifications and amendments to the SCIS system which may then be applied in some continuous better refinements such that SCIS can be further improved before the society really get into a conclusive decision about whether a true implementation.

Conclusion & Limitations of HKLam Statistical Model Theory

A few final conclusive words, this author understand that such (Chinese type of) Social Credit Information system may NOT be a good thing as the system is full of controversies about the pros and cons as stated aforementioned. This author does believe that the technology itself is actually neutral but its good or bad usage(s) are completely depending on the highest leadership of its finally completed country. That said, if the nuclear bomb technology was first invented by the axis of evil's dictatorship leaders in World War II but NOT the liberal & democratic U.S.A., then our free World history may be completely inverted from the middle to late last Century or even nowadays. We may still be ruled under a unified military dictatorship empire instead of a general open minded democratic (Western) World with all major common Western humanity values that are owned by lots of individual multi-nations. In reality, from our ancient human history until nowadays, those who want to have a unification of all nations over the world may still face his/her own "Battle of Waterloo" (weakness) even the most famous emperor, Napoleon, still cannot escape from exceptions (real world war). As in the

Eastern World, there was once a historic second large empire – the Mongoian Empire. According to the historical record, the area occupied by her acrossed over the both Asia and Europe. But as those Mongoian first conqueror's successors always had their own weakness. Hence, the Mongoian can only "Get the World through the horses" but they cannot "rule the World through the horses". The great and large Mongoian empire was then broken down into several small countries. In fact, ancient Chinese was once got her independence from the Mongoian central government after a series of revolutionary wars. One of the famous legend is about the Mid-Autumn Festival such that Chinese people may put a paper written encrypted statement inside the moon cakes to communicate about the revolutionary affairs. In brief, the China's independent war against the Mongoian may be an example for the hers double standard over the issue of independence. Hence, this author believes that it should be our God & Lord, Jesus Christ's will that every human established empire will have her own ending day except God's own forever one that will be established in the future heaven for all of His true believers together with followers. In some Western countries, people are now discussing about the possibility of some Western common human valued laws for the monitoring or a democratic elected committee without any political filtering (but Iran has her own filtering system that is worked against the Western common values) to oversee the implementation of such kind social credit system if it is unavoidable in some real world cases. Indeed, these laws and election are another story and may be presently out of the scope of this paper with left to those interesting academic professionals for the future research as my main focus is in the mathematical and statistical related aspects. By the way, there are still some limitations to my HKLam statistical model theory itself. One of them may be its error (vector) estimation and the associated recursive reduction for the next stepped random matrix generation until the final optimal value attended. This is because the whole algorithmic process is just an approximation and may depend on the linear regression-combination method such as the Gram-Schmidt orthogonalization process or a similar method as stated in [1]. Moreover, there may be a great difference between the original and the final related optimum random matrix and thus implies how may one choose a suitable error vector may be important. Indeed, the promise for the optimum regression-combination (or the random matrix) is just coming from the fixed point theorem or its associated one (the contraction theorem) for the linear transformation. However, it is well-known that the expected value for the set of all these error vectors (when you treat these errors as a random variable only) is just zero. Hence, we may still approximate the next level random matrix from its previous linear regression-combination which are correct without any loss of generality. In practice, the HKLam statistical model theory is correct as the statement has already been proved by the linear algebra mathematics method in the present review section or detail in [1].

Last but not least, there may be a need for a large scale of experimental data collection for the further optional verification to my HKLam statistical theory or even for the adjustment (i.e. fine tuning) of the theory as it is just a new developed theory like my first attained university in 1992, the Hong Kong University of Science & Technology which is a newly established one in 1991. To sum up, this author conclude that myself developed and proposed HKLam statistical model should be elementary said to be true or correct as it is proved qualitatively (pure mathematics with linear

algebra & linear regression) as well as quantitatively (the first 10 Riemann Zeta non-trivial zeros' numerical values with the Root model equation) in the previous review section of the present paper.

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