International Journal of Science Academic Research
Vol. 01, Issue 09, pp.612-618, December, 2020
Available online at http://www.scienceijsar.com

ISSN: 2582-6425

Research Article

THE NEUTRONS SCATTERING ANALYSIS BY ^{60}Cu, NUCLEUS USING VARIATIONAL MOMENT APPROACH

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Received 20th October 2020; **Accepted** 17th November 2020; **Published online** 15th December 2020

Abstract

Abstract: The variational moment approach for the neutrons scattering analysis by ^{60}**Cu**, nucleus within the energy range (60-80) MeV is applied to the construction of the complex single-particle mean field felt by neutrons in ^{60}**Cu**, starting from negative energy values to the positive energy values. The results according to the variational moment approach would contain: the continuous energy variations of the radial moments of the real and imaginary parts of the mean field, which are connected by dispersion relations, were compared with these resulting from global parameterization of the optical model potential, and the continuous energy variations of the volume and surface depths of the imaginary part of the mean field, also the continuous energy variations of the radius parameter of the Wood-Saxon approximation to the mean field potential, In addition to the continuous energy variation of the depth of the real potential obtained by adding dispersive correction with its Hartree-Fock approximation of the nonlocal potential and determining the behavior of the energy dependence of both two depths. Consequently, our results of the continuous energy variations of: the radial moments of the real and imaginary parts of the mean field showed the excellent agreement with these resulting from global parameterization of the optical model potential and with these resulting from the single fits of the potential parameters of the experimental data, the predicted total cross section within the energy range (10-153) MeV and elastic differential cross section for selected energies (60, 65 and 70) MeV showed the excellent agreement with available experimental data and better than these resulting from global parameterization of the optical model potential.

Keywords: Variational Moment Approach (VMA), Dispersion Relations (DR), Total Cross Section, Neutrons Scattering, Optical Neutron Potential, Mean Field, Fermi Energy.

INTRODUCTION

The nuclear optical model potential is of the fundamental importance concepts in the nuclear physics. It describes the motion of one nucleon, bound or unbound, in the mean field of all the other nucleons comprising the nucleus. The field due to the sum of all the individual nucleon-nucleon interactions is thus represented by a simple one-body potential. This approximation greatly simplifies the calculation of a wide range of nuclear structure and nuclear reaction phenomena, in addition to the excellent agreement with experimental data (Hodgson, 1990). The application of the concept of the nuclear mean field is for understanding the properties of bound single-particle states and for elastic scattering of unbound nucleons (Hodgson, 1990; Koning and Delaroche, 2003; Mahaux and Sartor, 1991).

The phenomenological optical model potential for nucleon-nucleus scattering, U, is defined as (Koning and Delaroche, 2003; Mahaux and Sartor, 1991; IAEA, 2006; Melkanoff *et al.*, 1961; Al-Mustafa and Belal, 2019 & 2020):

$$U(r,E) = -V_V(r,E) - V_{SO}(r,E).\vec{\sigma}.\vec{l} + V_d(r,E) + V_C(r) + i\left(-W_V(r,E) - W_d(r,E) + W_{SO}(r,E).\vec{\sigma}.\vec{l}\right) \quad (1)$$

Where $V_{V,S}$ and $W_{V,S,SO}$ are the real and imaginary components of the volume-central (V), surface-central (d) and spin-orbit (SO) potentials, respectively. $E$is the LAB energy of the incident particle in MeV. All components are separated in energy-dependent well depths, $V_V, V_d, V_{SO}, W_V, W_d$ and W_{SO}, and energy-independent radial parts f, namely

$$\begin{aligned} V_V(r,E) &= V_V(E) f(r,R_V,a_V) \\ W_V(r,E) &= W_V(E) f(r,R_V,a_V) \\ W_d(r,E) &= -4a_d W_d(E) \frac{d}{dr} f(r,R_d,a_d) \\ V_d(r,E) &= -4a_d V_d(E) \frac{d}{dr} f(r,R_d,a_d) \\ V_{SO}(r,E) &= V_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r,R_{SO},a_{SO}) \\ W_{SO}(r,E) &= W_{SO}(E) \left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} f(r,R_{SO},a_{SO}) \end{aligned} \quad (2)$$

The form factor $f(r,R_i,a_i)$ is a Wood-Saxon shape

$$f(r,R_i,a_i) = \frac{1}{[1+e^{(\frac{r-R_i}{a_i})}]} \quad (3)$$

Where the geometry parameters are the radius $R_i = r_i A^{\frac{1}{3}}$,, with A the atomic mass number, and the diffuseness parameters $a_i, i = V, SO, d$. For neutrons scattering, the value of the coulomb term V_C, is zero. By solving the Schrödinger equation numerically with this complex potential yields a wealth of valuable information; it returns a prediction for the basic observables, namely the elastic angular distribution and the reaction and total cross section (IAEA, 2006; Melkanoff *et al.*, 1961; Al-Mustafa and Belal, 2019 & 2020). The variational moment approach (VMA)describes the continuous energy variation of the constructed complex mean field and the radial moments (volume integral per nucleon)of their components felt by neutrons in ^{60}Cu which incorporates thedispersion relation that connects their real and imaginary parts, and reliable in an energy domain which typically extend from energies below to energies above the Fermi energy E_F. Moreover, the mean field is required to closely reproduce the experimental value of the Fermi energy.and so the reliable determination of the mean field is perfect by comparing a prediction of the cross sections with these are measured experimentally. There are many published studies for detailed

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analyses of data for the neutron scattering state, some of these studies depended on the single fits of the experimental data and others depended on dispersion relations. In both states our dependence is on global parametrization of the optical model potential which agree with the energy and atomic mass ranges of the ^{60}Cu nucleus. The present paper aims at presenting the variational moment approach (VMA) of the neutrons scattering by ^{60}Cu nucleus and comparing the results with these resulting from global parametrization of the optical model potential and available experimental data within energy range (60-80) MeV and its extend to the reliable low and high energy domain from the studied energy range according to evaluated fitting methodology.

METHODOLOGY

The methodology of (VMA) is summarized as follows (Koning and Delaroche, 2003; Mahaux and Sartor, 1991; IAEA, 2006; Melkanoff *et al.,* 1961; Al-Mustafa and Belal, 2019 & 2020; Romanovsky *et al.,* 1993 & 1995):

Volume integral per nucleon

Determining the continuous energy variation of the volume integral per nucleon by using Brown-Rho (Br) expression:

For the central imaginary part of the nuclear mean field:

$$[r^2]_W(E) = \beta \frac{(E-E_0)^2}{(E-E_0)^2+\rho_W{}^2} \quad (4)$$

The imaginary part has a volume and a surface component, the volume component is,

$$[r^2]_{W_V}(E) = \beta \frac{(E-E_0)^2}{(E-E_0)^2+\rho_{W_V}{}^2} \quad (5)$$

So, the surface component is,

$$[r^2]_{W_d}(E) = [r^2]_W(E) - [r^2]_{W_V}(E) \quad (6)$$

where $\beta, \rho_W, \rho_{W_V}$ denote Brown-Rho parameters, E_0 is:

$$E_0 = \frac{E_F}{2} \quad (7)$$

Where, E_F, the Fermi energy in MeV, that is defined as the energy halfway between the last occupied and the first unoccupied shell of the nucleus, determined from the experimental masses as follows (Wapstra and Gove, 1971):

$$E_F = \frac{E_F^+ + E_F^-}{2}$$
$$E_F^+ = M_{A+1} - M_A - m \quad (8)$$
$$E_F^- = M_A - M_{A-1} - m$$

Where E_F^+ is the negative of the separation energy of a nucleon from the (A+1)-nucleon system. Also, E_F^- is the negative of the separation energy of a nucleon from the A-nucleon system, m is the atomic mass of the incident particle.

Depths of the Volume and Surface Absorption of the Mean Field

Determining the continuous energy variation of the volume and surface absorption depths,

$$\mathrm{W_V(E)} = [r^2]_{W_V}(E)/\mathrm{g_{wv}}\,,\mathrm{MeV} \quad (9)$$

$$\mathrm{W_d(E)} = [r^2]_{W_d}(E)/\mathrm{g_{wd}}\,,\mathrm{MeV} \quad (10)$$

Where $\mathrm{g_{wv}}$,$\mathrm{g_{ws}}$can be written as follows:

$$g_{wv} = \frac{4\pi}{3}\frac{R_{wv}^3}{A_t{*}A_p}\left[1+\left(\frac{\pi a_{wv}}{R_{wv}}\right)^2\right] \quad (11)$$

$$g_{wd} = \frac{16\,\pi R_{wd}^2 a_{wd}}{A_t{*}A_p}\left[1+\frac{1}{3}\left(\frac{\pi a_{wd}}{R_{wd}}\right)^2\right] \quad (12)$$

Where $R_{wv} = r_{wv}A_t^{\frac{1}{3}}$, a_{wv}, $R_{wd} = r_{wd}A_t^{\frac{1}{3}}$, a_{wd} are the radius and diffuseness parameters of the volume and surface absorption.

Volume integral per nucleon of dispersive corrections of the real part of the mean field

The dispersion relations are a natural result of the causality principle that a scattered wave cannot be emitted before the arrival of the incident wave.

The dispersion component stems directly from the absorptive part of the potential,

$$\Delta\mathcal{V}(r,E) = \frac{\mathcal{P}}{\pi}\int_{-\infty}^{+\infty}\frac{\mathcal{W}(r,E')}{E'-E}dE' \quad (13)$$

Where $\mathcal{P}$ denotes the principal value. The total real central potential can be written as the sum of a Hatree-Fock term $\mathcal{V}_{HF}(r,E)$ and the total dispersion potential $\Delta\mathcal{V}(r,E)$

$$\mathcal{V}(r,E) = \mathcal{V}_{HF}(r,E) + \Delta\mathcal{V}(r,E) \quad (14)$$

Since $\mathcal{W}(r,E)$ has a volume and a surface component, the dispersive addition is,

$$\Delta\mathcal{V}(r,E) = \Delta\mathcal{V}_V(r,E) + \Delta\mathcal{V}_d(r,E)$$
$$= \Delta\mathcal{V}_V(E) f(r,R_V,a_V) - 4a_d\Delta\mathcal{V}_d(E)\frac{d}{dr}f(r,R_d,a_d) \quad (15)$$

Where the volume dispersion term is given by

$$\Delta V_V(E) = \frac{\mathcal{P}}{\pi}\int_{-\infty}^{+\infty}\frac{W_V(E')}{E'-E}dE' \quad (16)$$

And the surface dispersion term is given by

$$\Delta V_D(E) = \frac{\mathcal{P}}{\pi}\int_{-\infty}^{+\infty}\frac{W_D(E')}{E'-E}dE' \quad (17)$$

In general, "(16)" & "(17)" cannot be solved analytically. However, under certain plausible conditions, analytical solutions exist. Under the assumption that the imaginary potential is symmetric with respect to the Fermi energy E_F

$$W(E_F - E) = W(E_F + E) \quad (18)$$

Where W denotes either the volume or surface term, we can rewrite the dispersion relation as,

$$\Delta V(E) = \frac{2}{\pi}(E-E_F)\mathcal{P}\int_{E_F}^{\infty}\frac{W(E')}{(E'-E_F)^2-(E-E_F)^2}dE' \quad (19)$$

Determining the continuous energy variation of the volume

integral per nucleon of dispersive corrections of the real part of the mean field is obtained by using the dispersion relations:
The total dispersive correction:

$$[r^2]_{\Delta V_W}(E) = \frac{2}{\pi}(E - E_F)\int_{E_0}^{\infty} \frac{[r^2]_W(E')}{(E'-E_F)^2-(E-E_F)^2}dE' \quad (20)$$

The volume dispersive correction:

$$[r^2]_{\Delta V_{WV}}(E) = \frac{2}{\pi}(E - E_F)\int_{E_0}^{\infty} \frac{[r^2]_{W_V}(E')}{(E'-E_F)^2-(E-E_F)^2}dE' \quad (21)$$

So, the surface dispersive correction is:

$$[r^2]_{\Delta V_{Wd}}(E) = [r^2]_{\Delta V_W}(E) - [r^2]_{\Delta V_{WV}}(E) \quad (22)$$

Depths of the dispersive corrections of the real optical potential

Determining the continuous energy variation of the depths of the dispersive corrections of the real optical potential:

The volume dispersive correction:

$$\Delta V_V(E) = [r^2]_{\Delta V_{WV}}(E)/g_{wv}\text{, MeV} \quad (23)$$

The surface dispersive correction:

$$\Delta V_d(E) = [r^2]_{\Delta V_{Wd}}(E)/g_{wd}\text{, MeV} \quad (24)$$

So, the total dispersion potential $\Delta\mathcal{V}(r,E)$ calculated from "(15)", at $r = 0$.

Depth of the total real central potential

Determining the continuous energy variation of the depth of the total real central potential:

We determine the depth from "(14)", at $r = 0$,

Assumption that the Hatree-Fock term has a Wood-Saxon radial shape with energy-independent geometrical parameters (r_{HF}, a_{HF}) is given by

$$\mathcal{V}_{HF}(r,E) = \mathcal{V}_{HF}(E)f(r,R_{HF},a_{HF}) \quad (25)$$

Where the depth $\mathcal{V}_{HF}(E)$ is given by the following parametrization:

$$\mathcal{V}_{HF}(E)=\mathcal{V}_{HF}(E_F)e^{[\alpha_{HF}(E-E_F)/\mathcal{V}_{HF}(E_F)]}E\geq E_F$$
$$\mathcal{V}_{HF}(E)=\mathcal{V}_{HF}(E_F)+\alpha_{HF}(E-E_F)E\leq E_F \quad (26)$$

Where α_{HF}, the slope parameter, $R_{HF} = r_{HF}A^{1/3}$, radius parameter, $\mathcal{V}_{HF}(E_F)$ is the depth at Fermi energy.

Volume integral per nucleon of the real potential

Determining the continuous energy variation of the volume integral per nucleon of the real potential:

The volume integral per nucleon of the real potential is given by:

$$[r^2]_V(E) = [r^2]_{HF}(E) + [r^2]_{\Delta V_W}(E) \quad (27)$$

Where $[r^2]_{HF}(E)$, the volume integral per nucleon of the Hartree-Fock that can be written as follows,

$$[r^2]_{HF}(E) = \mathcal{V}_{HF}(E) * g_{HF} \quad (28)$$

Where g_{HF}, is given by

$$g_{HF} = \frac{4\pi}{3}\frac{R_{HF}^3}{A_t*A_p}\left[1+\left(\frac{\pi a_{HF}}{R_{HF}}\right)^2\right] \quad (29)$$

Radius parameter of the total real central potential

Determining the continuous energy variation of the radius parameter of the Woods-Saxon approximation to the full potential.

We determine the radius parameter of the Woods-Saxon approximation to the full optical potential from the equation:

$$R_V(E)^3 + (\pi a_V)^2R_V(E) - \left(\frac{3}{4\pi}\right)g_V(E)A_tA_p = 0 \quad (30)$$

Where a_V, diffuseness parameter and $g_V(E)$, can be determined from the relation:

$$g_V(E) = [r^2]_V(E)/\mathcal{V}(E) \quad (31)$$

So, the radius parameter will be:

$$r_V(E) = R_V(E)A^{-1/3} \quad (32)$$

Comparing with the global parameterizations of the optical model potential

After calculating the volume integral per nucleon of the mean field components, we have compared them with global parameterizations of the optical potential, in addition to calculating the depths and the geometrical parameters whose calculations have been performed in the (VMA) program:

1. Koning and Delaroche (Kd) [2], for
   $0.001 \leq E \leq 200$ Mev, $Z_t = (12-83)$, $A_t = (24-209)$
2. Madland (Md) [11,15], for
   $50 \leq E \leq 400$ Mev, $Z_t = (6-82)$, $A_t = (12-208)$

Also, we have compared our results with these global parameterizations for the continuous energy variations of the predicted total cross sections, in addition to elastic differential cross section for the energy values (60, 65 and 70) MeV and within the angular range of the center-of-mass scattering angle $(2° - 172°)$whose calculations have been performed in the (SPI-GENOA) program (Perey, 1975).

RESULTS AND DISCUSSION

The results According to the (VMA) and (SPI-GENOA) programs are summarized as follows:

Input Parameters

The values of the input parameters in the VMA program for the neutrons scattering by ^{60}Cu nucleus are showed in the (Table 1).

Table 1. The values of the input parameters

Brown-Rho Parameters					
ρ_w, MeV		ρ_{W_V}, MeV		β, MeV. fm^3	E_f (MeV)
11.0		58.0		93.0	-10.8848128
Geometrical Parameters					
a_V, fm	r_{wv}, fm	r_d, fm	a_{wv}, fm	a_d, fm	
0.664	1.261	1.261	0.602	0.602	
Hartree-Fock Parameters					
r_{HF}, fm	a_{HF}, fm	α_{HF}	$\mathcal{V}_{HF}(E_F)$, MeV		
1.236	0.62	0.448	492.59		
(Spin- Orbit) term Parameters					
V_{SO}, fm	W_{SO}, fm	r_{SO}, fm	a_{SO}, fm		
6.8	0.0	1.2	0.6		
(Projectile-Target) Parameters					
Z_n	A_n(amu)[14]	Z_t	A_t (amu) [14]		
0.0	1.0086	29	59.594		

Volume integrals per nucleon of the imaginary parts of the mean field

The energy dependence of the volume integrals per nucleon of the imaginary parts of the mean fields are compared with these resulted from global parameterizations of the optical potential and with these resulting from the single fits of the potential parameters of the experimental data according to SPI program, within the energy range $(E_f - 120)$MeV, as they are showed in the Figure (1). From the figure it becomes clear for us: The energy dependence of the volume integrals per nucleon showed agreement in the behavior comparing with these resulted from global parameterization of the optical model potential, and fitting of these resulted from the single fits of the available experimental data.

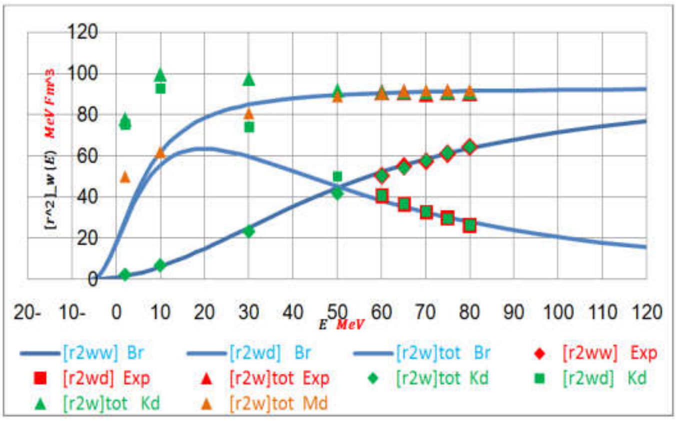

Figure 1. Volume integrals per nucleon of the imaginary parts of the mean field as a function of neutron energy (the red line) compared with these resulted from global parameterization of the optical model potential and with these resulted from the single fits of the potential parameters of the experimental data.

Depths of the imaginary parts of the mean field

The energy dependence of the depths of the (volume and surface) imaginary parts of the mean field within the energy range (from -100 to +100) MeV are showed in the Figure (2). From the figure we have observed a rapid variation of the depths in the vicinity of the Fermi energy and slowly variation toward the highly energies which are ascribed to a strong coupling between the elastic channel and the other reaction channels.

Volume integral per nucleon of total dispersive correction of the real part of the mean field

The energy dependence of the volume integral per nucleon of total dispersive correction of the real part of the mean field within the energy range from (-100 to +100) MeV is showed in the Figure (3).

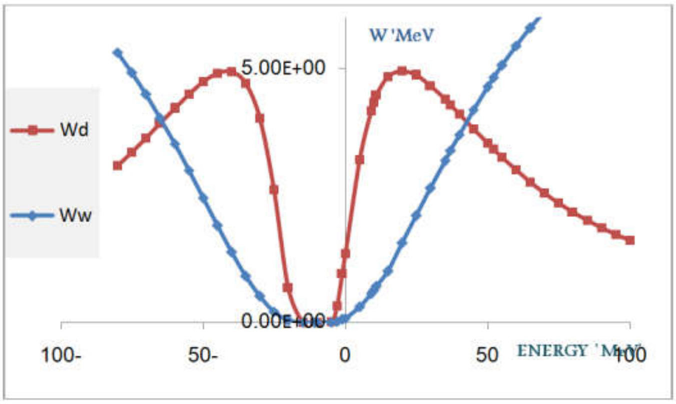

Figure 2. Depths of the (volume and surface) imaginary parts of the mean field as a function of neutron energy

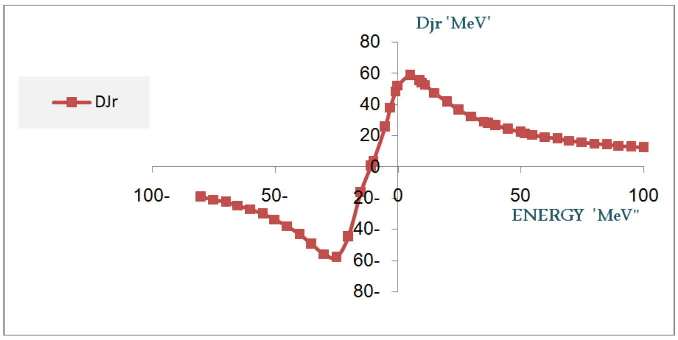

Figure 3. Volume integral per nucleon of total dispersive correction of the real part of the mean field as a function of neutron energy

Volume integral per nucleon of the real part of the mean field

The energy dependence of the volume integral per nucleon of the real part of the mean field obtained using dispersion relations with its HF approximation of the nonlocal potential for bound and unbound energies are compared with these resulted from global parameterizations of the optical potential and with these resulted from the single fits of the potential parameters of the experimental data (Jeukenne and Mahaux, 1986) according to SPI program. as they are showed in the Figure (4).

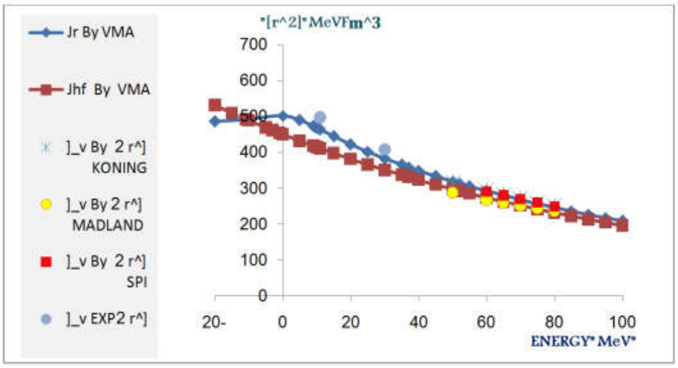

Figure 4. Volume integral per nucleon of the real part of the mean field as a function of neutron energy compared with these resulted from global parameterization of the optical model potential and with these resulted from the single fits of the potential parameters of the experimental data

Depth of the total real central potential

The energy dependence of the depth of the total real central potential obtained by adding dispersion correction with its HF approximation of the nonlocal potential for bound and unbound energies are showed in the Figure (5).

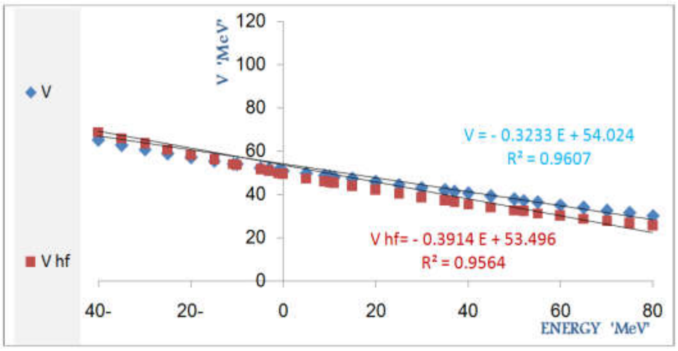

Figure 5. The energy dependence of the depth of the Wood-Saxon approximation to the mean field potential with its HF approximation

From the figure it becomes clear for us: The energy dependence behavior of both two potentials are the linear behavior according to the two following equations:

$$U_V(E) = -0.3233\,\mathrm{E} + 54.024 \quad (33)$$

$$U_{HF}(E) = -0.3914\,\mathrm{E} + 53.496 \quad (34)$$

The real radius parameter of the mean field

The energy dependence of the real radius parameter of the Wood-Saxon approximation to the mean field potential within the energy range from -80 MeV to 110 MeVis showed in the Figure (6). From the figure we have observed a rapid variation of the real radius parameter (a characteristic wiggle) in the vicinity of the Fermi energy and then slow variation toward the high energies. This wiggle is thus due to a strong coupling between the elastic channel and the other reaction channels.

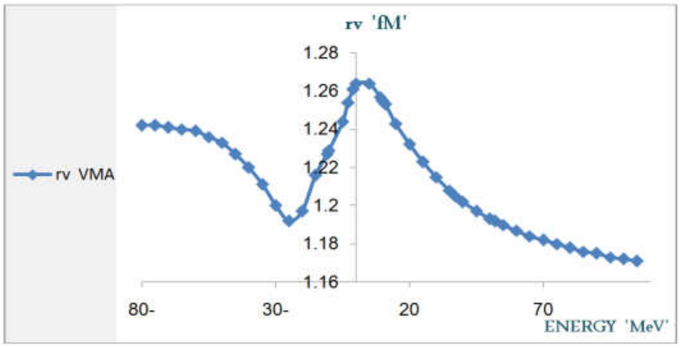

Figure 6. The energy dependence of the radius parameter of the Wood-Saxon approximation to the mean field potential with its HF approximation

Cross sections

The total cross section within the energy range $(5 - 153)$ MeV is compared with these resulted from global parameterizations of the optical potential and with available experimental data (Taylor and Wood, 1941; TENDL, 2019), and are (mb), as they are showed in the Figure (7). There is excellent agreement with the experimental data and the global parameterization of the optical potential according to our calculations in the (SPI-GENOA) program.

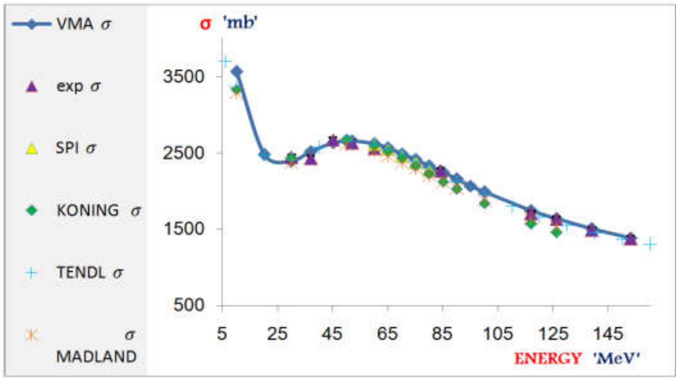

Figure 7. The energy dependence of the total cross section (the red line) compared with experimental value and with these resulted from global parameterization of the optical model potential

Elastic differential cross sections and polarization for selected energy

The elastic differential cross sections for selected energies whose magnitude (60, 65 and 70) MeV compared with these resulted from global parameterizations of the optical potential, which are showed in the Figures (8-10).

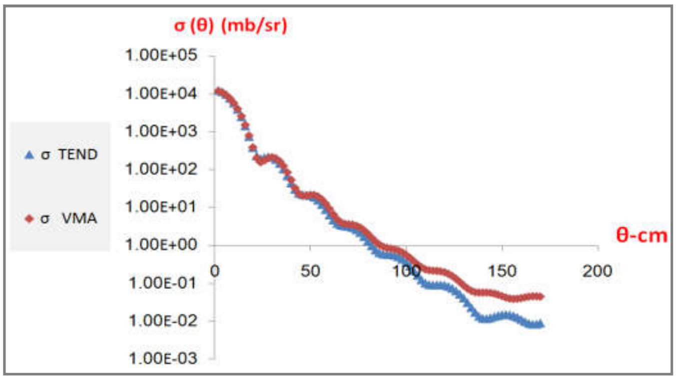

Figure 8. Dependence of the elastic differential cross section upon the center-of-mass scattering angle (the red line) compared with these resulted from global parameterization of the optical model potential, for $E_{Lab} = 60\ MeV$

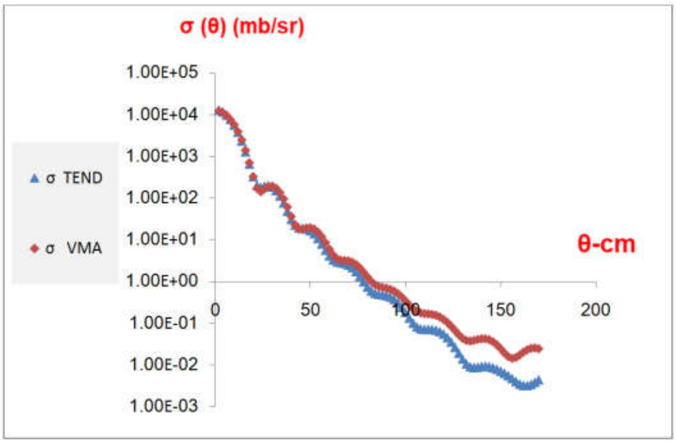

Figure 9. Dependence of the elastic differential cross section upon the center-of-mass scattering angle (the red line) compared with these resulted from global parameterization of the optical model potential, for $E_{Lab} = 65\ MeV$

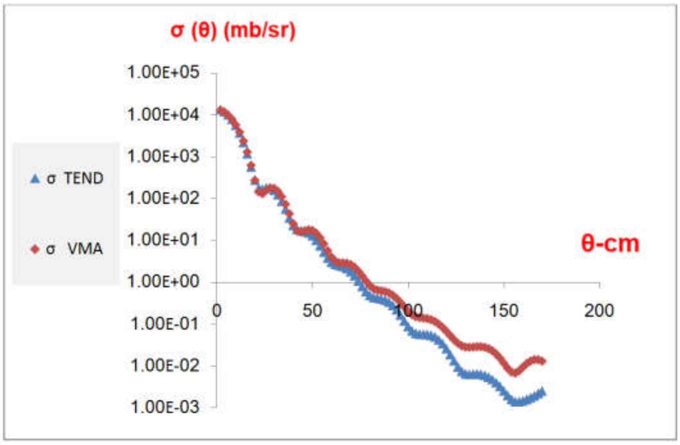

Figure 10. Dependence of the elastic differential cross section upon the center-of-mass scattering angle (the red line) compared with these resulted from global parameterization of the optical model potential, for $E_{Lab} = 70\ MeV$

There is an excellent agreement with the global parameterization of the optical model potential (TENDL, 2019) according to our calculations in the (SPI-GENOA) program.

Conclusion

The important conclusions can be shown as follows:

i. Our analysis of the neutrons scattering by ^{60}Cu nucleusaccording to the variational moment approach drawn for certain input values of the mean field parameters.
ii. Our calculation of the continuous energy variations of the volume integrals per nucleon of the imaginary parts of the mean fields showed an excellent agreement with these resulted from global parameterizations of the optical potential and with these resulted from the single fits of the potential parameters of the experimental total cross sections data.
iii. Our calculation of the continuous energy variation of the depths of the (volume and surface) imaginary parts of the mean field for bound and unbound energies showed excellent agreement in the behavior (symmetric) in the vicinity of the Fermi energy.
iv. Our calculation of the continuous energy variation of the volume integral per nucleon of the real part of the mean field obtained by adding dispersion correction with its HF approximation of the nonlocal potential for bound and unbound energies showed an excellent agreement with these resulted from global parameterizations of the optical potential and with these resulted from the single fits of the potential parameters of the experimental total cross sections data.
v. Our calculation of the continuous energy variation of the depth of the real part of the mean field obtained by adding dispersion correction with its HF approximation of the nonlocal potential for bound and unbound energies showed the energy dependence behavior of both two potentials are the linear behavior according to the two equations (33-34). In addition to continuous energy variation of the real radius parameter of the Wood-Saxon approximation to the mean field potential is a characteristic wiggle in the vicinity of the Fermi energy. This wiggle is thus due to a strong coupling between the elastic channel and the other reaction channels.
vi. Our prediction of the total cross section data within the energy range $(5 - 153)$ MeV showed excellent agreement with available experimental data and the better than these resulted from global parameterization of the optical model potential.
vii. Our prediction of the elastic differential cross section and polarization data for selected energies (60,65 and 70) MeV within the angular range $\theta_{cm} = (2° - 172°)$, showed excellent agreement with these resulted from global parameterization of the optical model potential. and thus more reliable for calculation the cross sections of unknown interactions of elements nuclei and their isotopes such as neutrons scattering by titanium element nucleus and its natural isotopes.

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